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INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK-DEPENDENT CONSUMPTION RATE

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ABSTRACT: In this paper we successfully provide a rigorous and efficient method to derive the optimal solution for the inventory models with deteriorating items and multi dependent consumption rate. We have taken a more realistic demand rate that depends on two factors, one is time, and the other is the stock level. The stock level in itself obviously gets depleted due to the customer's demand. As a result, what we witness here is a circle in which the customer's demand is being influenced by the level of stocks available, while the stock levels are getting depleted due to the customer's demands.

KEYWORDS: Inventory model, customer demand

INTRODUCTION

In the literature of inventory after the development of classical economic order quantity (EOQ) model researchers extensively studied several aspects of inventory modeling by assuming constant demand rate. But in a real market demand of a product is always dynamic state due to the variability of time, price or even of the instantaneous level of inventory displayed in retail shop. This impressed researchers and marketing practitioners to think about the variability of demand rate.

REVIEW OF LITERATURE

Silver and Peterson (1985) also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Gupta and Vrat (1986) first developed a model to minimize the cost with the assumption that stock-dependent consumption rate is a function of the initial stock level. Padmanabhan and Vrat (1988) considered stock-dependent consumption rate as a function of inventory level at any instant of time. Vrat *et al.* (1990) developed a model to determine optimum ordering quantity for stock dependent consumption rate items under inflationary environment with infinite replenishment rate without permitting shortages. Datta *et al.* (1991) made an attempt to investigate the effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate without shortages. Pal, S., Goswami, A., Chaudhuri, K.S. (1993) studied a deterministic inventory model for deteriorating items with stock-dependent demand rate. Urban, T.L. (1995) considered an inventory model with the demand rate dependent on stock and shortages levels. Hariga, M.A. (1995) deals an effect of inflation and time value of money on an inventory model with time-varying demand rate. Shortages are also permitted in this model. Padmanabhan and Vrat (1995) further presented inventory models for perishable items with stock-dependent selling rate. The selling rate is assumed to be a function of current inventory level. Ray *et al.* (1997) developed a finite time-horizon deterministic economic order quantity (EOQ) inventory model with shortages under inflation, where the demand rate at any instant depends on the on-hand inventory (stock level) at that instant.

SUPPLY CHAIN MANAGEMENT

Supply chain management (SCM) is the management of a network of interconnected businesses involved during the product and service packages required by end customers. Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point of origin to point of consumption (supply chain). "design, planning, execution, control, and monitoring of supply chain activities with the objective of creating net value, building a competitive infrastructure, leveraging worldwide logistics, synchronizing supply with demand and measuring performance globally". Supply chain management, is a set of organizations directly linked by one or more of the upstream and downstream flows of products, services, funds, and information from a source to a customer.

A supply chain is a network of interconnected facilities which performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.

Supply chain management is the integration of key business processes across the supply chain for the purpose of creating value for customers and stakeholders. A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.

Many manufacturing operations are designed to maximize throughput and lower costs with little consideration for the impact on inventory levels and distribution capabilities. Purchasing contracts are often negotiated with very little information beyond historical buying patterns. The result of these factors is that there is not a single, integrated plan for the organization---there were as many plans as businesses. Clearly, there is a need for a mechanism through which these different functions can be integrated together. Supply chain management is a strategy through which such integration can be achieved.

Supply chain management is typically viewed to lie between fully vertically integrated firms, where the entire material flow is owned by a single firm and those where each channel member operates independently. Therefore coordination between the various players in the chain is key in its effective management.

ROLE OF INVENTORY IN SUPPLY CHAIN

Inventory is held throughout the supply chain in the form of raw materials, work in process and finished goods. Inventory exists in the supply chain because of a mismatch between supply and demand. This mismatch is intentional at a manufacturer, where it is economical to manufacture in large lots that are then stored for future sales. The mismatch is also intentional at a retail store where inventory is held in anticipation of future demand. Inventory is a major source of cost in a supply chain and has a huge impact on responsiveness. An important role that inventory plays in the supply chain is

- To increase the amount of demand that can be satisfied by having the product ready and available when the customer wants it.
- To reduce cost by exploiting economics of scale that may exist during production and distribution.
- To support a firm’s competitive strategy. If a firm’s competitive strategy requires very high level of responsiveness, a company can achieve this responsiveness by locating large amounts of inventory close to a customer. Conversely, a company can also use inventory to become more efficient by reducing inventory through centralized stocking.

ASSUMPTIONS AND NOTATIONS

The mathematical models of an inventory problem are based on the following assumptions and notations:

(1) The consumption rate D(t) at any time t is assumed to be $\alpha + \beta\, I(t) + \gamma\, t\text{-}ds$, where α is a positive constant, β is the stock-dependent demand rate parameter, $0 \le \beta, \gamma \le 1$, and $I(t)$ is the inventory level at time t.

(2) The replenishment rate is infinite and lead time is zero.

(3) The planning horizon is finite.

(4) Shortages are backlogged at the rate of $e^{-\delta t}$ where $0 < \delta < 1$ and t is the waiting time for next replenishment.

(5) The deterioration rate is $a + b\, t$; $a, b > 0$.

(6) Product transactions are followed by instantaneous cash flow.

NOTATIONS

r discount rate, representing the time value of money

i inflation rate

R $r–i$, representing the net discount rate of inflation is constant

H	planning horizon
T	replenishment cycle
m	the number of replenishments during the planning horizon n = H/T
T_j	the total time that is elapsed up to and including the j^{th} replenishment cycle (j=1,2,…..n) where $T_0 = 0$, $T_1 = T$, and $T_n = H$.
t_j	the time at which the inventory level in the j^{th} replenishment cycle drops to zero (j=1,2,…..n)
$T_j – t_j$	time period when shortages occur (j=1,2,…..n)
Q	the 2^{nd}, 3^{rd},…..n^{th} replenishment lot size.
I_m	maximum inventory level
A	ordering cost per replenishment
C	per unit cost of the item
C_h	holding cost per unit per unit time
C_s	shortage cost per unit per unit time
C_o	opportunity cost per unit per unit time due to lost sale.

MODEL FORMULATION

Suppose the planning horizon H is divided into n equal intervals of length $T = H/n$. Hence the reorder times over the planning horizon H are $T_j = jT$ (j=1, 2,…..n). The period for which there is no shortage in each interval $[jT, (j+1)T]$ is a fraction of the scheduling period T and is equal to kT, $(0 < k < 1)$. Shortages occur at time $t_j = (k+j–1)T$, (j=1,2,…..n) and are accumulated until time $t=jT$ (j=1,2,…..n)and shortages are backlogged exponentially.

The first replenishment lot size of I_m is replenished at $T_0=0$. During the time interval $[0, t_1]$ the inventory level decreases due to stock-dependent demand rate and deterioration and falls to zero at $t = t_1$, now shortages start during the time interval $[t_1, T]$ and accumulated until $t = T_1$.

The differential equations governing the instantaneous state of inventory level at any time t are given by

$$I'(t) + (a+bt)\, I(t) = -[+\beta I(t)+\gamma t-ds] \text{ with } I(t_1)=0 \quad 0\ t\ t_1 \qquad \ldots(1.1)$$

$$I'(t) = -(\alpha + \gamma\, t-ds)\, e^{-\delta t} \quad t_1\ t\ T \qquad \ldots(1.2)$$

The respective solutions of the above differential equations are

$$I(t) = (\alpha - ds)\exp\left(-(a+\beta)t - \frac{bt^2}{2}\right)\int_t^{t_1}(\alpha - ds + \gamma x)\exp\left((a-ds+\beta)x + \frac{bx^2}{2}\right)dx \quad 0\ t\ t_1 \qquad \ldots(1.3)$$

And
$$I(t) = -\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - te^{-\delta t}\right)\right] \quad t_1\ t\ T \qquad \ldots(1.4)$$

The maximum inventory level during first replenishment cycle is

$$I(0) = I_m = \int_0^{t_1}(\alpha - ds + \gamma x)e^{\left((a-ds+\beta)t + \frac{bt^2}{2}\right)}dt \qquad \ldots(1.5)$$

And the maximum shortage quantity during the first replenishment, which is backlogged

$$I_b = \frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - Te^{-\delta T}\right)\right] \qquad \ldots(1.6)$$

The present value of the ordering cost during first replenishment cycle is A, as the replenishment is done at the start of each cycle.

The present value of the holding cost of inventory during first replenishment cycle is

$$H.\ C. = C_h\int_0^{t_1} I(t)e^{-Rt}dt \qquad \ldots(1.7)$$

The present value of the shortage cost during first replenishment cycle is

$$S.\ C. = C_s\int_{t_1}^{T}\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - te^{-\delta t}\right)\right] e^{-Rt}dt \qquad \ldots(1.8)$$

Replenishment items are consumed by demand as well as deterioration during $[0, t_1]$. The present value of material cost during the first replenishment cycle is

$$C_p = CI_m + \frac{Ce^{-RT}}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - Te^{-\delta T}\right)\right] \quad \ldots(1.9)$$

Opportunity cost due to lost sale

$$O.\ C. = C_o \int_{t_1}^{T} (\alpha - ds + \gamma t)\left(1 - e^{-\delta t}\right) e^{-Rt} dt \quad \ldots(1.10)$$

Consequently, the present value of total cost of the system during the first replenishment cycle is

$$TRC = A + H.C. + S.C. + C_p + O.C. \quad \ldots(1.11)$$

The present value of total cost of the system over a finite planning horizon H is

$$TC\ (m,\ k) = \sum_{j=1}^{m-1} TRC\ e^{-RjT} - Ae^{-RH} = TRC\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) - Ae^{-RH} \quad \ldots(1.12)$$

The present value of total cost TC (n, k) is a function of two variables n and k, where n is a discrete variable and k is a continuous variable. For a given value of n, the necessary condition for $TC\ (m,\ k)$ to be minimized is $\frac{dTC(m,k)}{dk} = 0$ which gives

$$\frac{C_h H}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\exp\left((a - ds + \beta)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^2\right)\int_0^{\frac{kH}{m}} \exp\left(-(a - ds + \beta)t - \frac{bt^2}{2}\right)e^{-Rt}dt$$

$$+\frac{C_s H}{\delta Rm}\left[(\delta\alpha - ds + \gamma)\left\{\exp\left(\frac{(\delta k + R)H}{m}\right) - \exp\left(\frac{(\delta + R)kH}{m}\right)\right\} - \gamma\left(1 - \frac{\delta kH}{m}\right)\right.$$

$$\left.\times\exp\left(-\frac{(\delta k + R)H}{m}\right) + \frac{\gamma}{(\delta + R)}\left\{\left\{\delta - \left(\frac{\delta(\delta + R)kH}{m} - R\right)\right\}\exp\left(-\frac{(\delta + R)kH}{m}\right)\right]$$

$$+\frac{CH}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\left[\exp\left((a - ds + \beta)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^2\right) - \exp\left(\frac{(\delta k + R)H}{m}\right)\right]$$

$$\frac{C_o H}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\left[\exp\left(-\frac{(\delta + R)kH}{m}\right) - \exp\left(-\frac{RkH}{m}\right)\right] = 0 \quad \ldots(1.13)$$

Provided the condition $\frac{d^2TC(m,k)}{dk^2} > 0$ is satisfied.

We follow the optimal solution procedure proposed by Montgomery (1982), we let (m^*, k^*) denote the optimal solution of $TC(m,\ k)$ and let $(m,\ k(m))$denote the optimal solution to $TC(m,\ k)$ when m is given. If $\tilde{m}$ is the smallest integer such that $TC(\tilde{m}, k(\tilde{m}))$is less than each value of $TC(\tilde{m}, k(\tilde{m}))$ in the interval $\tilde{m}+1 < \tilde{m} < \tilde{m}+10$. Then we take $(\tilde{m}, k(\tilde{m}))$ as the optimal solution to $TC(m,\ k(m))$. Hence $(\tilde{m}, k(\tilde{m})) = (m^*, k^*)$. Using the optimal procedure described above, we can find the maximum inventory level and optimal order quantity to be

$$I_m = \int_0^{\frac{k^*H}{m^*}} (\alpha - ds + \gamma x)\exp\left((a - ds + \beta)x + \frac{bx^2}{2}\right) dx \quad \ldots(1.15)$$

And $$Q^* = \int_0^{\frac{k^*H}{m^*}} (\alpha - ds + \gamma x)\exp\left((a - ds + \beta)x + \frac{bx^2}{2}\right) dx$$

$$+\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta k^* H/m^*} - e^{-\delta H/m^*}\right) + \delta\gamma\left(\frac{k^*H}{m^*}e^{-\delta k^* H/m^*} - \frac{H}{m^*}e^{-\delta H/m^*}\right)\right] \quad \ldots(1.16)$$

CONCLUSION

In this paper, to make our study more suitable to present-day market, we have done our research in an inflationary environment. Even till now, most of the researchers have been either completely ignoring the decay factor or are considering a constant rate of deterioration which is not practical. Therefore, we have taken the time dependent decay factor. The problem has been formulated analytically and has been used to arrive at the optimal solution. Also, we could extend the deterministic model into a stochastic model. Finally, we could generalize the model to allow for quantity discounts, trade credits, and others.

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